## MATH-8 TEST Unit 3 (Intro to Trig Identities, Equations and Inverse Trig) <br> SAMPLE <br> NAME:

100 points

- Notebook should be turned in before test. It will not be accepted after.
- Phones must be turned OFF and put away. Any visible phone (smart watch, headphones, ipad etc.) will result in a grade $F$.
- No scratch paper or notes.
- No graphing calculator.
- No credit will be given for solutions if work is not shown.
- I expect clear and legible presentations.

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, tear it off and place it at the front of your desk, I will collect it. Once you have turned in part one, you may not go back to it.

## PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following: (Note: answers to inverse trig. problems should be in radians, not degrees)
(a) $\sin ^{-1}\left(\frac{-1}{2}\right)=$ $\qquad$ (b) $\cos ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(c) $\cos ^{-1}\left(\frac{\sqrt{2}}{2}\right)=$ $\qquad$ (d) $\sin \left(\frac{2 \pi}{3}\right)=$ $\qquad$
(e) $\tan ^{-1}(\sqrt{3})=$ $\qquad$ (f) $\sin ^{-1}(1)=$ $\qquad$
(g) $\cos ^{-1}(2)=$ $\qquad$ (h) $=\tan ^{-1}\left(-\frac{\sqrt{3}}{3}\right)$
(2) HOW MANY solutions does each of the following equations with the given restrictions on $\theta$ have? (Do not need to solve, just tell how many solutions there would be.)
(a) $\sin \theta=-1 / 7 ; \quad 0 \leq \theta \leq 2 \pi$
(c) $\sin \theta=-1 / 7$ $\qquad$
(b) $\theta=\sin ^{-1}(-1 / 7)$ $\qquad$ (d) $\quad \sin \theta=-1 / 7 ; \quad 0 \leq \theta \leq \frac{\pi}{2}$
(3) The domain of $f(x)=\cos ^{-1} x$ $\qquad$
(4) The range of the function $f(x)=\sin ^{-1} x$ $\qquad$
(5) The period of the function $f(x)=\tan x$ is $\qquad$ .
(6) The vertical asymptotes of $f(x)=\sec (5 x)$ are located at: $\qquad$
$\qquad$
PART TWO - CALCULATORS ALLOWED (no graphing calc.)

Fill in the blanks. (2 points each)
(1) Give an identity for $\sin 2 \theta=$ $\qquad$
(2) Give an identity for $\cos (\alpha+\beta)=$ $\qquad$
(3) Give an identity for $\sin (q / 2)=$ $\qquad$
(4) What is domain of $f(x)=\cos ^{-1}(x)$ ? $\qquad$
(5) Find all asymptotes for $f(x)=3 \tan \left(\frac{\pi}{5} x\right)$
(6) What is the range of $f(x)=\tan ^{-1}(x)$ ? $\qquad$
(7) Solve for $0 \leq x<2 \pi$ : $\sin x=-1$
(8) Using identities, find the exact, simplified value of: ( points each) (you must show work, for credit)
(a) $\sin \left(\frac{7 \pi}{12}\right)$
(b) $\cos \left(157.5^{\circ}\right)$ $\qquad$
(9) Simplify exactly:
(a) $\cos \left(\sin ^{-1}(-2 / 5)\right)=$
(b) $\sin \left(2 \cos ^{-1}(1 / 4)\right)=$
(10) Sketch the following graph. (clearly show scale, graph at least one period, show location of any asympototes, label 2 points on graph)
$f(x)=-2 \sec \left(\frac{\pi}{3} x\right)$

(11) Sketch the following graph. (clearly show scale, graph at least one period, show location of any asympototes, label 2 points on graph)
$f(x)=4 \tan (4 x)$

(12) Given $\tan \alpha=2 / 3$, $\alpha$ in the third quadrant, and $\cos \theta=12 / 13, \frac{3 \pi}{2}<\theta<2 \pi$ Find:
a) $\sin (\alpha-\theta)$
b) $\cos (\theta / 2)$
c) $\tan (2 \alpha)$
(13) Prove the following identity. Presentation should be very clear

$$
\sin 2 \theta=\frac{2 \tan \theta}{1+\tan ^{2} \theta}
$$

(14) Given the following information about $\theta$,

Find
a) $\tan (2 \theta)$
b) $\cos \left(\frac{\theta}{2}\right)$
(15) Given that $P(-1,2)$ lies on the terminal side of $\theta$, find
a) $\cos (2 \theta)$.
b) $\theta$
(16) Solve the following equations exactly for $0 \leq \theta \leq 2 \pi$.
(a) $\cos \theta=\frac{1}{5}$
(b) $\sin \theta=0.8$
c) $\quad \tan \theta=-7$
(17) Solve for $0 \leq x<2 \pi$ : $\sqrt{3} \tan (2 x)+1=0$
(18) Find all solutions: $4 \cos \left(\frac{x}{3}\right)=-4$

Find all solutions to the following equations.
(19) $2 \sin ^{2}(x)-\cos (2 x)=0$
(20) $\quad 4 \cos ^{2} \mathrm{x}-2=0$

SOLVE the following equations: $0 \leq x<2 \pi$
(21) $\sin (3 x)-\sin (2 x)=0$
(22) $\sec ^{2} x-3 \tan x=-1$

## HELPFUL IDENTITIES: CAN TEAR OFF

$$
\begin{aligned}
& \sin \alpha \sin \beta=\frac{1}{2}[\cos (\alpha-\beta)-\cos (\alpha+\beta)] \\
& \cos \alpha \cos \beta=\frac{1}{2}[\cos (\alpha-\beta)+\cos (\alpha+\beta)] \\
& \sin \alpha \cos \beta=\frac{1}{2}[\sin (\alpha+\beta)+\sin (\alpha-\beta)]
\end{aligned}
$$

$$
\begin{aligned}
& \sin \alpha+\sin \beta=2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right) \\
& \sin \alpha-\sin \beta=2 \sin \left(\frac{\alpha-\beta}{2}\right) \cos \left(\frac{\alpha+\beta}{2}\right) \\
& \cos \alpha+\cos \beta=2 \cos \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right) \\
& \cos \alpha-\cos \beta=-2 \sin \left(\frac{\alpha+\beta}{2}\right) \sin \left(\frac{\alpha-\beta}{2}\right)
\end{aligned}
$$

